Techniques for Finding Derivatives

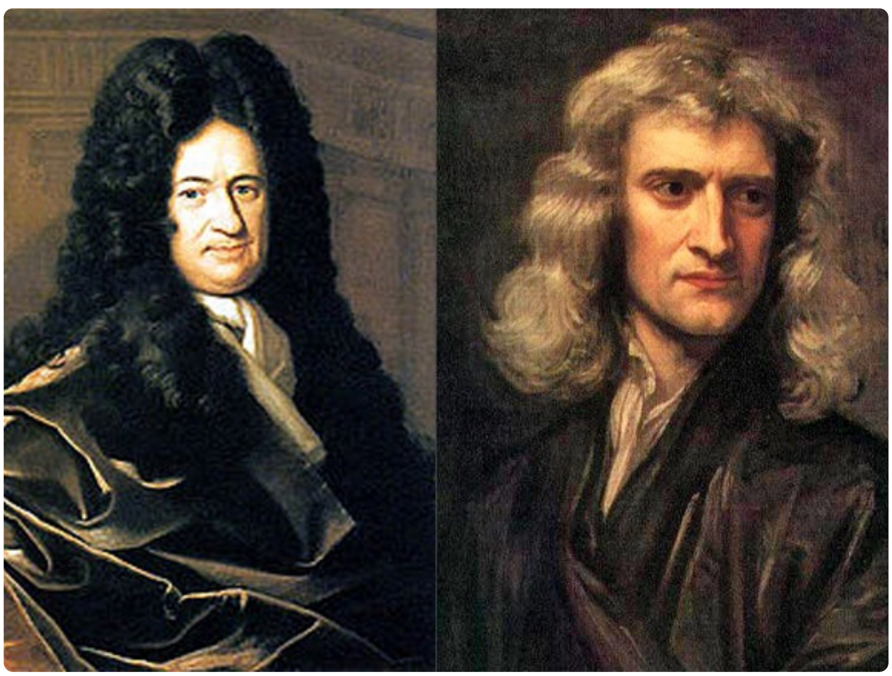
f - function

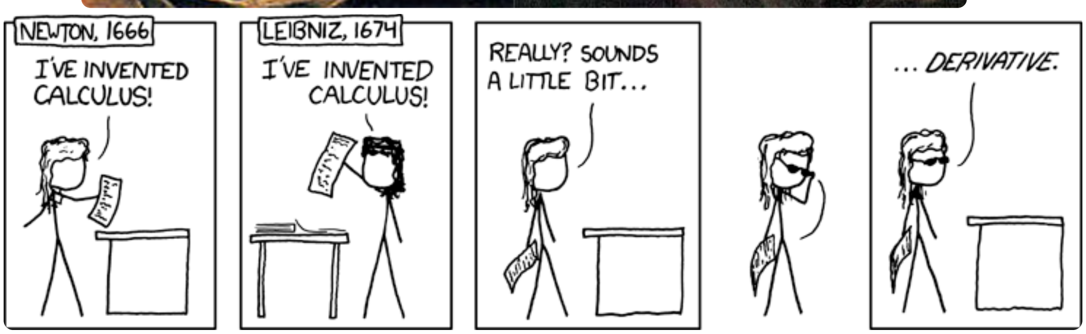
Notation : Derivative of f

$$\frac{df}{dx}, \quad \frac{dy}{dx}, \quad \frac{d(f(x))}{dx}, \quad \frac{d}{dx}(f(x)) \qquad\qquad f', \quad f'(x)$$

(Warning : Not a fraction !!!) (f prime)

Leibniz Newton





Basic Rules of Differentiation (Can be used without using limit definition):

1/ (Constant Rule)

constant function $f(x) = k$

$$\boxed{\frac{d(k)}{dx} = 0}$$

$\leftarrow$ zero function

2/ (Power Rule) For any number r

$$\boxed{\frac{d x^r}{dx} = r x^{r-1}}$$

3/ (Constant Multiple rule) For any number k

$$\boxed{\frac{d\,(k f(x))}{dx} = k \frac{d(f(x))}{dx}}$$

4/ (Sum / Difference Rule)

$$\boxed{\frac{d\,(f \pm g)}{dx} = \frac{df}{dx} \pm \frac{dg}{dx}}$$

<u>Example</u> $f(x) = 3x^2 + \sqrt{x} - 4 \Rightarrow \frac{df}{dx} = ?$

$$\frac{df}{dx} = \frac{d}{dx}(3x^2 + x^{1/2} - 4)$$

Sum / Difference Rule

$$= \frac{d}{dx}(3x^2) + \frac{d}{dx}(x^{1/2}) - \frac{d}{dx}(4)$$

Constant Multiple rule

$$= 3\frac{d}{dx}(x^2) + \frac{d}{dx}(x^{1/2}) - \frac{d}{dx}(4)$$

$$= 3 \cdot 2x^{2-1} + \frac{1}{2}x^{(\frac{1}{2}-1)} - 0$$

Power Rule and constant rule

$$= 6x + \frac{1}{2}x^{-\frac{1}{2}}$$

<u>Example</u> $f(x) = (\sqrt{x}+2)(x+1) \Rightarrow \frac{df}{dx} = ?$

$$f(x) = x^{3/2} + 2x + x^{\frac{1}{2}} + 2$$

All of our rules

$$\Rightarrow \frac{df}{dx} = \frac{3}{2}x^{3/2-1} + 2 \cdot 1 \cdot x^{1-1} + \frac{1}{2}x^{\frac{1}{2}-1} + 0$$

$$= \frac{3}{2}x^{\frac{1}{2}} + 2 + \frac{1}{2}x^{-\frac{1}{2}}$$

Profit = Revenue − cost

<u>Example</u> $P(x) = R(x) - C(x)$

$$\Rightarrow P'(x) = R'(x) - C'(x)$$

Marginal Profit = Marginal Revenue − Marginal cost

<u>Marginal Analysis</u>

Let $C(x) = 2x + 3$ (linear cost function)

$$\Rightarrow C(x+1) - C(x) = (2(x+1)+3) - (2x+3)$$

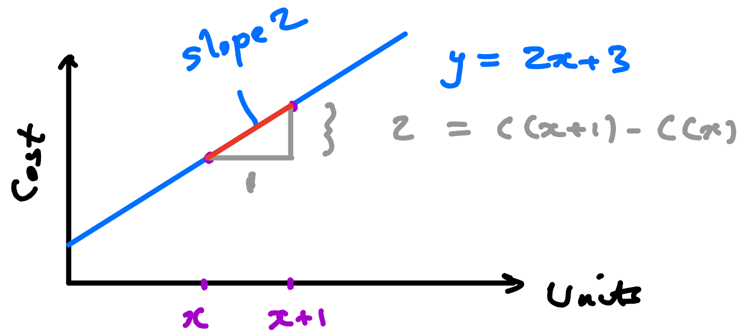
$$= 2x + 2 + 3 - 2x - 3$$

$$= 2$$

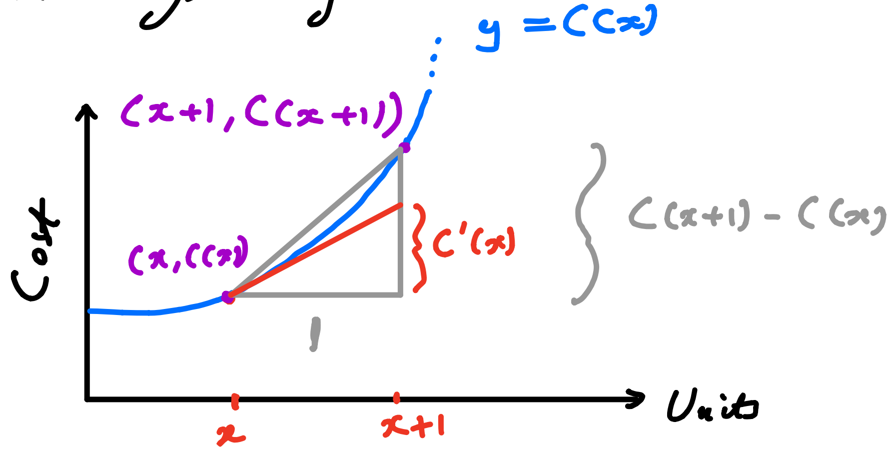
Increase in cost making one more unit

marginal cost.

Picture :



More generally :



$$\Rightarrow \quad C'(x) \approx C(x+1) - C(x)$$

Only an approximation. Best if $y = C(x)$ is close to linear.

$$\Rightarrow \quad \text{Marginal cost at } x \approx C(x+1) - C(x)$$

Same holds for marginal revenue / profit.

Example If the profit from making/selling 10 units of a product is \$100 and the marginal profit at 10 is 6, estimate the profit from making / selling 11 units.

$$P'(10) = 6 \approx P(11) - P(10) \Rightarrow P(11) \approx 6 + P(10) = 106$$

Revenue and Demand

q = number of units that will sell (we've also used x before)

p = price per unit

Demand equation : $p = D(q)$ ← number of units that will sell at price p

$$\Rightarrow \quad R(q) = pq = D(q)q.$$

revenue from selling q units

for every unit sold you make p

Example (q = number of units to be made / sold)

A product is to be made and sold. The cost function is linear with marginal cost 3 and fixed cost 1. If the demand equation is $p = 4 - q$ determine the marginal profit function (a function in q).

marginal cost

$$C(q) = 3q + 1 \leftarrow \text{fixed cost}$$

$$R(q) = pq = (4-q)q = 4q - q^2$$

$$\Rightarrow \quad P(q) = R(q) - C(q) = 4q - q^2 - 3q - 1$$

$$= q - q^2 - 1$$

$$\Rightarrow \quad P'(q) = 1 - 2q \quad \text{(Differentiation rules)}$$